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Connective Instability in Binary reactive Viscoelastic Fluids with chemical Reactions in a saturated Porous Medium

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ABSTRACT:
This research investigates the effect of chemical reaction on the onset of convective instability of binary viscoelastic fluid saturated porous layer. The study employs linear stability theory combined with the normal mode technique to analyze the system's stability. The governing momentum equation is established using the modified Darcy-Oldroyd model, which accounts for time derivative. Furthermore, we derive analytical expressions (solutions) for the critical conditions that define both stationary (steady) and oscillatory (time-dependent) convection. The findings demonstrate that both viscoelasticity and chemical reactions significantly influence the stability of the system, either delaying or promoting the onset of convection depending on the governing parameters. The effect of the Prandtl number (Pr), solute Rayleigh number, diffusivity ratio (τ), relaxation and retardation parameters, chemical reaction parameter on the stability of the system is investigated and are shown graphically.
KEYWORDS: *Convective instability, viscoelastic fluid, chemical reaction, porous medium, stability analysis, heat and mass transfer.*

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I. INTRODUCTION

Recent interest in thermosolutal convection in porous media has been motivated by its wide range of applications, in geophysics, chemical engineering, petroleum recovery, environmental sciences, and materials processing. The onset of convection is often governed by thermal gradients, concentration differences, and the rheological characteristics of the fluid. A comprehensive review of the literature concerning the natural convection in porous fluid-saturated porous media may be found in the books by Nield and Bejan [1], Bortola and Cafaro [2], Ingham and Pop [3,4], Vafai [5,6], and Vadaz [7].

The study of convection in porous media originated with the pioneering work of Horton and Rogers[8], and Lapwood [9], who investigated thermal instability in fluid-saturated porous layers. Their analyses established the fundamental criteria for the onset of convection in porous structures and formed the basis for subsequent theoretical developments.

Chandrasekhar [10], provided a comprehensive treatment of hydrodynamic stability theory and laid the groundwork for analyzing convective instabilities in fluid systems. Later studies incorporated concentration gradients into convection models, leading to the development of double-diffusive convection theory. Turner [11] demonstrated the importance of thermal and solutal effects in determining fluid stability and transport characteristics.

Investigations by Steinberg and Brand [12] demonstrated that reactive transport processes introduce additional coupling mechanisms that significantly affect convective onset.

Furthermore, Nield [13] examined the onset of thermohaline convection in a porous medium.

Larson [14], examined the thermal instabilities in viscoelastic flows in horizontal porous layers as initial problem. A study by Rosenblat [15] discussed the thermal convection of a viscoelastic fluid. Recently studies on connective instability for binary viscoelastic fluids [16,17], and viscoelastic fluid in porous media [18], can be found in the literature. Recently, Swamy, Naduvinamani and Sidram [19], investigated the Onset of Darcy-Brinkman covectionin binary viscoelastic fluid saturated porous layer.

The study of viscoelastic fluid convection emerged from the need to model non-Newtonian fluids encountered in industrial and biological applications. Researchers such as Walters [20], Joseph[21], and Sharma,et.al [22,23] examined the effects of fluid elasticity on hydrodynamic stability and showed that viscoelastic parameters can modify critical Rayleigh numbers and convection patterns. Subsequent

investigations extended these analyses to porous media and revealed that fluid elasticity may either delay or promote instability depending on the constitutive model employed.

The influence of chemical reactions on convective systems has also received considerable attention. Studies have shown that reaction kinetics can alter concentration gradients and either stabilize or destabilize fluid layers depending on reaction rates and diffusivity ratios.

Postelnicu [24] studied the Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media. Rudraiah and Malashetty [25], Malashetty and Gaikwad [26] made important contributions to the understanding of double-diffusive convection and chemically reactive flows in porous media. Their studies demonstrated that reaction rates, permeability, and solutal gradients strongly influence the onset of convection. More recent investigations have considered viscoelastic fluid models, including Oldroyd-B and Maxwell fluids, in porous environments and reported significant deviations from Newtonian predictions.

Recently, Malashetty et.al [27, 28] investigated chemically driven instabilities in binary liquid mixtures with fast chemical reactions and derived analytical conditions for both stationary and oscillatory instabilities. A study by Wang and Tan [29], discussed the Stability analysis of double-diffusive convection of Maxwell fluid in a porous medium heated from below.

Despite substantial progress, relatively few studies have examined the simultaneous effects of viscoelasticity, binary diffusion, and chemical reactions in porous media. The complex interaction among these mechanisms remains an active area of research. The present work contributes to this field by analyzing the onset of convective instabilities in chemically reactive binary viscoelastic fluids and assessing the influence of key governing parameters on system stability.

II. PROBLEM FORMULATION AND MATHEMATICAL ANALYSIS

An infinite horizontal porous layer of thickness *h*, saturated with a binary viscoelastic fluid, is considered. The porous medium is bounded by two horizontal planes located at z = 0, and z = *h*. The lower boundary is maintained at a higher temperature $T_0 + \Delta T$ and solute concentration $S_0 + \Delta S$, while the upper boundary is kept at temperature T_0, and concentration S_0, thereby establishing destabilizing thermal and stabilizing solutal gradients across the layer. A Cartesian coordinate system is adopted with the origin at the lower boundary and the z-axis directed vertically upward, while gravity force ***g***, acts vertically downward on the system. Fluid motion within the porous medium is described by the modified Darcy law appropriate for an Oldroyd-B viscoelastic fluid, which accounts for both relaxation and retardation effects. Under these assumptions, the governing equations for mass conservation, momentum, energy, and solute transport are formulated as follows.

$$\nabla.\mathbf{V} = 0, \tag{1}$$

$$\left(\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{V}}{\partial t} - \rho g + \nabla p\right)\left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right) = \frac{\mu}{K}\left(1 + \bar{\lambda}_2 \frac{\partial}{\partial t}\right)\mathbf{V}, \tag{2}$$

$$\frac{\partial T}{\partial t} = -\frac{1}{\gamma}\left((\mathbf{V}.\nabla)T - (\kappa_T \nabla^2)T\right), \tag{3}$$

$$\frac{\partial C}{\partial t} + \frac{1}{\varepsilon}\left(\mathbf{V}.\nabla - \kappa_C \nabla^2\right)C = \frac{k}{\varepsilon}\left(C_{eq}(T) - C\right), \tag{4}$$

$$\rho_b = \rho_0[1 - \alpha_T(T_b - T_0) + \alpha_C(C_b - C_0)], \tag{5}$$

where $\boldsymbol{V}$ denote the velocity vector, k, is the lumped reaction rate , p is the pressure, $\bar{\lambda}_1$ is the relaxation time depending on viscoelasticity, $\bar{\lambda}_2$ is the retardation time due to the action of the porous matrix, K is the permeability o the porous medium, T is the temperature , C is the concentration, γ is the ratio of the heat capacities, α_T , α_c , and κ_T κ_C are thermal, solute expansion co-efficient , thermal and solute diffusivities, respectively. $C_{eq}(T)$ is the equilibrium concentration of the solute at a given temperature.

Following the Pritchard and Richardson [14], it is assumed that the equilibrium solute concentration is a linear function of temperature so that $C_{eq}(T) = C_0 + \varphi(T - T_0)$.

Further, if the chemical equilibrium at the boundaries assumed, $\varphi = \dfrac{\Delta C}{\Delta T}$. The coefficient φ could be positive or negative. If φ > 0, the solubility increases with temperature, While if φ < 0, the solubility decreases with temperature. It should be noted that only the case $\varphi > 0$ is considered in this paper.

2.1 Basic state

The unperturbed state is presumed to be tranquil and is defined as

$u = v = w = 0, \quad T = T_b(z), \quad C = C_b(z), \quad \rho = \rho_b(z)$. (6)

which adhere to following equations,

$$\frac{dp_b(z)}{dx} = -\rho_b g, \quad \frac{d^2C_b(z)}{dz^2} = 0, \quad \frac{d^2T_b(z)}{dz^2} = 0, \tag{7}$$

$$\rho = \rho_0[1 - \beta_T(T - T_0) + \beta c(C - C_0)], \tag{8}$$

Then the solutions of the conduction state temperature $T_b(z)$, solute concentration $C_b(z)$, are given by

$$T_b = T_0 - \frac{\Delta T}{h}, \qquad C_b = C_0 - \frac{\Delta C}{h}, \tag{9}$$

2.2 Perturbed state

To investigate the stability of the system, the basic state is slightly disturbed by velocity, temperature, and solute are given by

$$\mathbf{V} = \mathbf{V}_b + \mathbf{V}', \qquad T = T_b + T_0', \qquad C = C_b + C_0' \tag{10}$$

Here prime indicate the perturbation quantity and assumed to be tiny. Then using Eq.(10) into Eqs.(1) - (5) with the Eqs,(6) - (9), we obtain the perturbed expressions as,

$$\left(\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{V}'}{\partial t} - \rho' g + \nabla p'\right)\left(1 + \bar{\lambda}_1 \frac{\partial}{\partial t}\right) = \frac{\mu}{K}\left(1 + \bar{\lambda}_2 \frac{\partial}{\partial t}\right)\mathbf{V}', \tag{11}$$

$$\frac{\partial T'}{\partial t} + \frac{1}{\lambda}\left(V.\nabla - \kappa_T \nabla^2\right)T' - w\frac{\Delta T}{h} = 0, \tag{12}$$

$$\frac{\partial C'}{\partial t} + \frac{1}{\varepsilon}\left(\mathbf{V}'.\nabla - \kappa_C \nabla^2\right)C' = \frac{k}{\varepsilon}\left(C_{eq}(T') - C'\right) \tag{13}$$

$$\rho = -\rho_0\left(\alpha_T T' - \alpha_C C'\right)_0, \tag{14}$$

By operating curl twice on Eq.(12), we eliminating the pressure term from the above equations and applying the Boussinesq approximation, the linearized governing equations for the velocity w, temperature T, and concentration C are obtained as follows (ignoring dashes).

$$\left(\lambda_1 \frac{\partial}{\partial t} + 1\right)\left(\frac{Da}{\varepsilon \,\mathrm{Pr}}\frac{\partial}{\partial t}\nabla^2 W - Ra_T \nabla_1^2 T + Ra_S \nabla_1^2 C\right) + \left(\lambda_2 \frac{\partial}{\partial t} + 1\right)\nabla^2 W = 0, \tag{15}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2 + V.\nabla\right)T - W = 0, \tag{16}$$

$$\left(\frac{\partial}{\partial t} - \tau.\nabla^2 + V.\nabla\right)C - W - \chi\left(T - C\right) = 0, \tag{17}$$

where $Ra_T = \dfrac{\alpha_T g \Delta T dk}{\upsilon \kappa_T}$, $Ra_S = \dfrac{\alpha_C g \Delta S dk}{\upsilon \kappa_T}$, $\lambda_1 = \dfrac{\kappa_T}{d^2}\bar{\lambda}_1$, $\lambda_2 = \dfrac{\kappa_T}{d^2}\bar{\lambda}_2$, $Da = \dfrac{k}{d^2}$,

$\mathrm{Pr} = \dfrac{\upsilon}{\kappa_T}$, $\tau = \dfrac{\kappa_S}{\kappa_T}$ the diffusivity ratio.

So with boundary conditions;

$$W = T = C = 0, \quad \text{at } Z = 0,\ 1. \tag{18}$$

III. LINEAR STABILITY ANALYSIS

Using the normal mode analysis, the perturbation variables *W, T,* and *C* are represented in the following form.

$$\begin{pmatrix} W \\ T \\ C \end{pmatrix} = \begin{pmatrix} w(z) \\ \theta(z) \\ \phi(z) \end{pmatrix} \exp(i(lx+my)+\omega t)\ , \tag{19}$$

Then substituting these expressions from Eq.(19) into Equations (15) to (17) and applying the principle of exchange of stabilities gives the governing equations,

$$(1+\omega\lambda_1)\left(\frac{\omega}{\mathrm{Pr}_D}(D^2-a^2)W + a^2 Ra_T\theta - a^2 Ra_S\phi\right) + (1+\omega\lambda_2)\left(D^2-a^2\right)W = 0 \tag{20}$$

$$W - \left(\omega - D^2 + a^2\right)\theta = 0, \tag{21}$$

$$-W\left(\omega - \tau(D^2-a^2)\right)\phi - \chi\left(\theta-\phi\right) = 0 \tag{22}$$

Assuming the most unstable solutions of the Eqs. (20) to (22), satisfying the boundary conditions (18) are taking in the form as,

$$\left(W(z),\Theta(z),\Phi(z)\right) = \left(W_0(z),\Theta_0(z),\Phi_0(z)\right) Sin(\pi z), \tag{23}$$

i.e., substituting Eq,(23) into Eqs.(20)-(),we obtain the corresponding matrix form of Eq.as

$$\begin{pmatrix} \left\{\frac{\omega}{\mathrm{Pr}_D} + \left(\frac{\omega\lambda_1+1}{\omega\lambda_2+1}\right)\right\}\delta^2 & -a^2\,\mathrm{Ra}_T & a^2\,\mathrm{Ra}_S \\ -1 & \omega+\delta^2 & 0 \\ -1 & -\chi & \chi+\omega+\tau\delta^2 \end{pmatrix} \begin{pmatrix} W_0 \\ \theta_0 \\ \phi_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{24}$$

Since, for non-trivial solution of Eq.(24), the thermal Rayleigh number is obtained as follows.

$$Ra_T = \frac{\delta^2}{a^2}\left(\frac{1+\lambda_2\omega}{1+\lambda_1\omega} + \frac{\omega}{\mathrm{Pr}\,D}\right)(\delta^2+\omega) + \frac{\mathrm{Ra_S}(\delta^2+\chi+\omega)}{\tau\delta^2+(\omega+\chi)}\ , \tag{25}$$

Where, $\delta^2 = (\pi^2 + a^2)$ is the total wave number. The expression of Eq.(25), gives a Rayleigh number can be used to analyse the stability in a binary viscoelastic fluid under the influence of chemical reaction.

3.1 **Stationary Mode.**

Under the principle of exchange of stabilities, the marginally stable stationary state ($\omega = \sigma_r + i\sigma_i = 0$), the Rayleigh number, Ra_T from Eq.(25), yields the following expressions for thermal Rayleigh number,

$$Ra_T^{St} = \frac{\delta^4}{a^2} + \frac{(\delta^2+\chi)}{\tau\delta^2+\chi} Ra_S\ , \tag{26}$$

In the absence of chemical reaction parameter, $\chi = 0$. Then Eq. (26) simplifies to the classical result.

$$Ra_T^{St} = \frac{\delta^4}{a^2}, \tag{27}$$

which has the minimum value of $4\pi^2$ occurs at $a_c = \pi$, then the classical result obtained by Horton and Rogers [8] and Lapwood [9] for Darcy porous media.

3.2. Oscillatory mode

For oscillatory convection, $\Delta_2 = 0$ ($\omega \neq 0$) in Eq. (25), then Ra_T can be written in the form:

$$Ra_T = \Delta_1 + i\sigma_i \ \Delta_2 . \tag{28}$$

where

$$\Delta_1 = \frac{\delta^2}{a^2}\left(\frac{\delta^2(1+\lambda_1(\lambda_2\sigma^2)}{1+\lambda_1^2\sigma^2} - \sigma^2\left(\frac{1}{\mathrm{Pr}_D} + \frac{\lambda_2-\lambda_1}{1+\lambda_1^2\sigma^2}\right)\right) + \frac{(\delta^2+\chi)(\tau\delta^2+\chi)+\sigma^2)}{(\delta^2\tau+\chi)^2+\sigma^2} Ra_S \ ,$$

$$\Delta_2 = \frac{\delta^2}{a^2}\left(\frac{a^2 Ra_S(\tau-1)}{\tau\delta^2+\chi)^2+\sigma^2} + \frac{\delta^2}{\mathrm{Pr}_D} + \frac{1-\delta^2\lambda 1+\delta^2\lambda 2+\lambda 1\lambda 2\sigma^2}{1+\lambda_1^2\sigma^2}\right) \ .$$

As Ra_T is a physical quantity, it must be real. So, it follows from the Eq.(28) that either $\sigma_i = 0$ or $\Delta_2 = 0$. Thus, a dispersion relation of the following type is obtained (by dropping subscript i)

$$r_1(\sigma_i^2)^2 + r_2(\sigma_i^2) + r_3 = 0, \tag{29}$$

with

$$r_1 = \delta^2\lambda_1(\delta^2\lambda_1 + \mathrm{Pr}_D\,\lambda_2) ,$$

$$r_2 = \delta^2(\mathrm{Pr}_D + \delta^2 - \mathrm{Pr}_D\,\delta^2\lambda_1 + \mathrm{Pr}_D\,\delta^2\lambda_2 - a^2\,\mathrm{Pr}_D\,Ra_S\lambda_1^2(-1+\tau) + \lambda_1(\mathrm{Pr}_D\,\lambda_2 + \lambda_1\delta^2)(\delta^2\tau+\chi)^2) ,$$

$$r_3 = \delta^2(-a^2\,\mathrm{Pr}_D\,Ra_S(-1+\tau) - (-\delta^2 + \mathrm{Pr}_D(-1+\delta^2(\lambda_1-\lambda_2)))(\delta^2\tau+\chi)^2) .$$

For $\Delta_2 = 0$, Eq. (28) takes the form:

$$Ra_T^{Osc} = \frac{\delta^2}{a^2}\left(\frac{\delta^2(1+\lambda_1(\lambda_2\sigma^2)}{1+\lambda_1^2\sigma^2} - \sigma^2\left(\frac{1}{\mathrm{Pr}_D} + \frac{\lambda_2-\lambda_1}{1+\lambda_1^2\sigma^2}\right)\right) + \frac{(\delta^2+\chi)(\tau\delta^2+\chi)+\sigma^2)}{(\delta^2\tau+\chi)^2+\sigma^2} Ra_S , \tag{30}$$

The oscillatory Rayleigh number obtained from Equation (30) is numerically minimized with respect to the wavenumber. Using the wavenumber determined from Equation (29), the influence of the governing physical parameters on the onset of oscillatory convection is systematically examined.

IV. NONLINEAR STABILITY ANALYSIS

In this section, we look at nonlinear analysis with a reduced Fourier series that only includes two terms. Although the linear stability analysis is sufficient for determining the stability condition of the motionless solution and the corresponding Eigen functions that qualitatively describe the convective flow, it cannot provide information about the values of the convection amplitudes or the rate of heat and mass transfer. To gain this additional information, we undertake nonlinear analysis, which is important for understanding the physical mechanism with the least amount of mathematical analysis and is a step toward comprehending the entire non-linear system.

To simplify the analysis, the flow is assumed to be two-dimensional, implying that all physical variables are independent of the y-direction. A stream function ψ, is introduced to satisfy the continuity equation, with the velocity components defined as u = ∂ψ/∂z, and

w = −∂ψ/∂x, substituting these expressions into Eq. (12), eliminating the pressure term, and applying the non-dimensional transformations given in Eq. (16) to the resulting equation together with Eqs. (13) and (14), the governing equations can be expressed in the following non-dimensional form:

$$\left(\lambda 1\frac{\partial}{\partial t}+1\right)\left(\frac{1}{\mathrm{Pr}_D}\frac{\partial(\nabla^2.\psi)}{\partial t} + Ra_C\frac{\partial C}{\partial x} + Ra_T\frac{\partial T}{\partial x}\right) + \left(1+\lambda 2\frac{\partial}{\partial t}\right)\nabla^2.\psi = 0, \tag{31}$$

$$\frac{\partial\psi}{\partial x} + \left(-\nabla^2 + \frac{\partial}{\partial t}\right)T + \frac{\partial(\psi,T)}{\partial(x,z)} = 0 , \tag{32}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{q}.\nabla - \frac{1}{Le}\nabla^2\right)C - w - \chi(T-C) = 0 , \tag{33}$$

A minimal double Fourier series representation for finite-amplitude convection is given by

$\Psi = A_{11}(t)Sin(ax)\ Sin(\pi z)$, (34)

$T = B_{11}(t)Cos(ax)\ Sin(\pi z) + B_{12}(t)\ Sin(2\pi z)$, (35)

$C = E_{11}(t)Cos(ax)Sin(\pi z) + E_{12}(t)Sin(2\pi z)$, (36)

Here, $A_{11}, B_{11}, B_{12}, E_{11}, E_{12}$ denote the time-dependent amplitudes, and are to be determined by the dynamics of the system. By using Eqs. (30) – (36) into the coupled nonlinear partial differential equations (31) – (33) and equating the coefficients of like terms, the following nonlinear system of ordinary differential equations is obtained.

$$\frac{dA11}{dt} = G, \tag{37}$$

$$\frac{dG}{dt} = -\frac{a\,\mathrm{Pr}_D}{\delta^2}\left(Ra_T\frac{dB_{11}}{dt} - Ra_C\frac{dE_{11}}{dt}\right) - \frac{\mathrm{Pr}_D}{\lambda_1}A_{11} + \frac{\mathrm{a\,Pr}_D}{\lambda_1\delta^2}\left(-Ra_T B_{11} + Ra_C E_{11}\right) - \left(\frac{1+\lambda_2\,\mathrm{Pr}_D}{\lambda_1}\right)G, \tag{38}$$

$$\frac{dB_{11}}{dt} = -aA_{11} - \delta^2 B_{11} - \pi a A_{11}B_{12}, \tag{39}$$

$$\frac{dB_{12}}{dt} = \frac{\pi}{2}aA_{11}B_{11} - 4\pi^2 B_{12}, \tag{40}$$

$$\frac{dE_{11}}{dt} = -aA_{11} - \pi a A_{11}E_{12} - \chi B_{11} - \left(\frac{\delta^2}{Le} + \chi\right)E_{11}, \tag{41}$$

$$\frac{dE_{12}}{dt} = \frac{\pi}{2}aA_{11}B_{11} + \chi B_{12} - \left(\frac{4\pi^2}{Le} + \chi\right)\mathrm{E}_{12}, \tag{42}$$

The non-linear truncated system is generally not solvable in closed form due to its strong nonlinear coupling; hence we have to obtain its solution using numerical method. Nevertheless the truncated system of Eqs.(37) to (42) remains uniformly bounded and retains the essential properties of the governing equations. The reduced system of Eqs.(37) to (42) must be dissipative. Thus, the phase-space volume must contract. In order to show the volume contraction, we must prove that the velocity field has a constant negative divergence indeed,

$$\frac{\partial A'_{11}}{\partial A_{11}} + \frac{\partial B'_{11}}{\partial B_{11}} + \frac{\partial B'_{12}}{\partial B_{12}} + \frac{\partial E'_{11}}{\partial E_{11}} + \frac{\partial E'_{12}}{\partial E_{12}} + \frac{\partial G'}{\partial G} = -\left((4\pi^2+\delta^2)(\tau+1) + 2\chi + \left(\frac{1+\lambda_2\,\mathrm{Pr}_D}{\lambda_1}\right)\right), \tag{43}$$

Since the divergence of the associated vector field is strictly negative, the reduced dynamical system is necessarily bounded and dissipative, where an over dash denotes differentiation with respect to time. As a consequence, all trajectories in the phase space undergo continuous volume contraction and ultimately approach an attracting invariant set of zero measure. Depending on the governing parameter values, this attractor may correspond to a stable equilibrium state, a periodic orbit, or a chaotic (strange) attractor. Then from Eq. (42), if a collection of initial conditions occupies a phase-space volume, *V(0)* at *t = 0*, then the corresponding volume at any later time , t is given by

$$V(t) = V(0)\ \boldsymbol{exp}\ \left(-\left\{\left(\delta^2+4\pi^2\right)\left(1+\frac{1}{Le}\right) + 2\chi + \left(\frac{1+\lambda_2\,\mathrm{Pr}\,D}{\lambda_1}\right)\right\}t\right), \tag{44}$$

This Eq.(44) indicates that the phase-space volume decays exponentially with time. The retardation parameter , Prandtl number, and diffusivity ratio further strengthen the dissipation. The system of Eqs.(37)-(42) also preserve its dynamical structure under the corresponding symmetry transformation.

We now derive the steady-state solutions analytically. Under the stationary condition, Eq.(37)-(42) reduce to following closed form expressions

$$G = 0, \tag{45}$$

$$\delta^2 A_{11} + Ra_T aB_{11} - Ra_C aE_{11} = 0, \tag{46}$$

$$aA_{11} + \delta^2 B_{11} + \pi a A_{11}B_{12} = 0, \tag{47}$$

$$-\frac{\pi}{2}aA_{11}B_{11}+4\pi^2B_{12}=0\ , \tag{48}$$

$$aA_{11}+\pi aA_{11}E_{12}-\chi B_{11}+(\frac{\delta^2}{Le}+\chi)E_{11}=0\ , \tag{49}$$

$$\frac{\pi}{2}aA_{11}E_{11}+\chi B_{12}-(\tau 4\pi^2+\chi)\,\mathrm{E}_{12}=0, \tag{50}$$

Now expressing the amplitudes $B_{11}, B_{12}, E_{11},$ and E_{12} in terms of A_{11}, substituting Eqs.(47)-(50) in Eq.(46), with $\frac{A_{11}^2}{8}=k$, and $Le=\frac{1}{\tau}$, we get

$$d_1k^2+d_2k+d_3=0\,, \tag{51}$$

The required root of Eq. (51) is given by

$$\frac{A_{11}^2}{8}\ =\frac{1}{2d_1}\left(-d_2+\sqrt{d_2^2-d_1d_3}\right)\ , \tag{52}$$

where $d_1 = 4a^4\mathrm{Le}^2\pi^2\delta^2,$

$d_2 = 4a^4\mathrm{Le}\pi^2(\mathrm{RaS}-\mathrm{LeRaT})+a^2\delta^2\left(\mathrm{Le}\chi(\delta^2+\mathrm{Le}\chi)+4\pi^2\left((1+\mathrm{Le}^2)\delta^2+\mathrm{Le}\chi\right)\right),$

$d_3 = (4\pi^2+\mathrm{Le}\chi)\left(\delta^6+\mathrm{Le}\delta^4\chi+a^2\left((\mathrm{LeRaS}-\mathrm{Rt})\delta^2+\mathrm{Le}(\mathrm{RaS}-\mathrm{RaT})\chi\right)\right).$

The threshold onset criterion for steady finite amplitude steady convection is obtained by letting the radical term in Eq.(52) to vanish. Hence the resulting finite-amplitude Rayleigh number takes the form

$$Ra_{_T}^{f}=\frac{1}{2y_1}[-y_2+\sqrt{y_2^2-4y_1y_3}\,], \tag{53}$$

Where $y_1 = 16a^8\mathrm{Le}^4\pi^4,$

$y_2 = 8a^6\mathrm{Le}^2\pi^2\left(-4a^2\mathrm{Le}\pi^2\mathrm{Rs}+\delta^2\left(\mathrm{Le}\chi(\delta^2+\mathrm{Le}\chi)+4\pi^2(\delta^2-\mathrm{Le}^2\delta^2+\mathrm{Le}\chi)\right)\right),$

$y_3 = 6a^8\mathrm{Le}^2\pi^4\mathrm{Rs}^2+a^4\delta^4\left(\mathrm{Le}\chi(\delta^2+\mathrm{Le}\chi)+4\pi^2(\delta^2-\mathrm{Le}^2\delta^2+\mathrm{Le}\chi)\right)^2-$
$8a^6\mathrm{Le}\pi^2\mathrm{Rs}\delta^2\left(4\pi^2\left((-1+\mathrm{Le}^2)\delta^2+\mathrm{Le}(-1+2\mathrm{Le})\chi\right)+\mathrm{Le}\chi\left((-1+2\mathrm{Le}^2)\delta^2+\mathrm{Le}(-1+2\mathrm{Le})\chi\right)\right),$ and $\tau=\frac{1}{Le}$.

The obtained amplitudes provide the necessary information for determining the heat and mass transfer characteristics of the convective system. Such characteristics play a crucial role in assessing convection in porous media, since the development of convective instability with increasing Rayleigh number is accompanied by a modification of the heat and solute fluxes. In the reference basic state, no convective transport occurs, and heat and mass are transferred exclusively by conduction and molecular diffusion. We next proceed to find the Nusselt number, and Sherwood number.

If and represent the rates of heat and mass transfer per unit area, respectively. Then these transport rates can be expressed as follows,

$$H = -\ \kappa_T\left\langle\frac{\partial Ttotal}{\partial z}\right\rangle_{z=0},\ \text{and}\qquad J = -\ \kappa_C\left\langle\frac{\partial Ctotal}{\partial z}\right\rangle_{z=0}, \tag{54}$$

Where the angular bracket corresponds to horizontal average and

$$Ttotal=T_0-\Delta T\frac{z}{d}+T(x,z,t),\ Ctotal=C_0-\Delta C\frac{z}{d}+C(x,z,t)\,. \tag{55}$$

Using Eqs.(35) and (36) in Eqs.(55) , and then substituting the resultant forms of equations in Eqs.(54). We obtain as

$$H=\kappa_T\frac{\Delta T}{d}\left(1-2\pi B_{12}\right)\ ,\ \text{and}\ J=\kappa_C\frac{\Delta C}{d}\left(1-2\pi E_{12}\right), \tag{56}$$

Writing B_{12}, E_{12} in terms of A_{11}, using Eqs.(45)-(50), and substituting in to Eqs.(56).
The Nusselt and Sherwood numbers are respectively defined by

$$\text{Nu} = 1+\left(\frac{2a^2A^2}{(8\delta^2+a^2A^2)}\right), \tag{57}$$

$$Sh = 1+2\pi\left(\frac{a^2A^2Le[a^2A^2\pi^2\,\text{Le}+8\pi^2Le(\delta^2+\chi)+2\chi(\delta^2+\chi Le)]}{\pi(8\delta^2+a^2A^2)[a^2A^2\pi^2Le^2+8\pi^2(\delta^2+\chi Le)+2(Le\chi(\delta^2+\chi Le))]}\right), \tag{58}$$

with $\text{k} = \frac{A_{11}^2}{8}$. The second terms on RHS of Eqs.(57) and (58) characterize the convective transport of heat and mass , respectively.

V. RESULTS AND DISCUSSION

The neutral stability curves for the oscillatory mode are presented in Figs. 1–4 to examine the effects of the solute Rayleigh number (Ra_s), solute diffusivity ratio (τ), stress relaxation time (λ_1), and strain retardation time (λ_2) on the onset of double-diffusive convection. The minimum point of each neutral curve represents the critical thermal Rayleigh number (Ra_T^c) and the corresponding critical wave number. Also, the influence of Darcy-Prandtl number (Pr_D), chemical reaction parameter (χ), solute Rayleigh number Ra_s), and the critical Rayleigh number under the λ_2-mode are illustrated in Figs. 3–6 and Figs. 1a–2b. The minimum point of each neutral stability curve corresponds to the critical thermal Rayleigh number (Ra_T^c) at which oscillatory convection first appears.

Figure 1 illustrates the influence of the solute Rayleigh number (Ra_s). It is observed that increasing $Ra_s$from 20 to 200 shifts the neutral curves upward, resulting in higher critical Rayleigh numbers. This indicates that stronger solutal buoyancy suppresses the onset of oscillatory convection and delays instability. Therefore, increasing Ra_shas a stabilizing effect on the system. Figure 2 depicts the effect of the solute diffusivity ratio (τ). As τincreases from 0.2 to 0.8, the neutral curves move downward, leading to lower critical Rayleigh numbers. This demonstrates that larger values of τpromote the onset of convection, indicating that the solute diffusivity ratio destabilizes the system. Physically, enhanced solute diffusion reduces the stabilizing concentration gradient, making convection easier to initiate.

Figure 3 shows the variation of the stress relaxation time (λ_1). It is evident that increasing $\lambda_1$from 0.5 to 2.0 raises the neutral curves and increases the critical Rayleigh number.

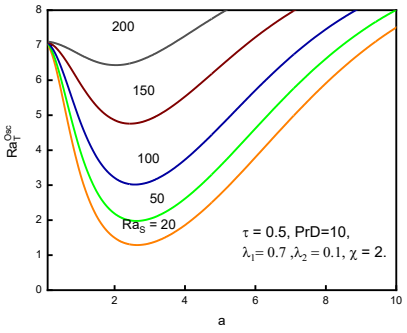


Fig1.Neutral curves for oscillatory mode for different values of solute Rayleigh number,RaS

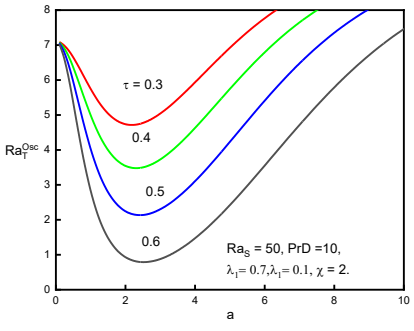


Fig2. Neutral curves for oscillatory mode for different values of solute diffusity ratio τ .

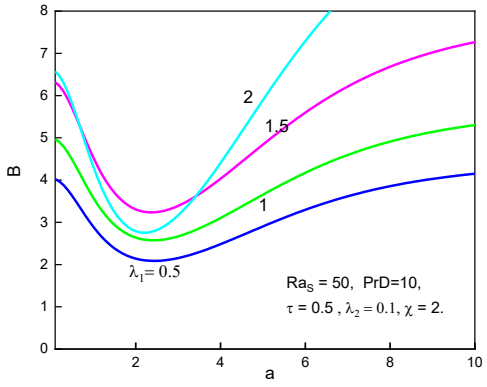


Fig 3. Neutral curves for oscillatory mode for different values of stress relation time λ_1.

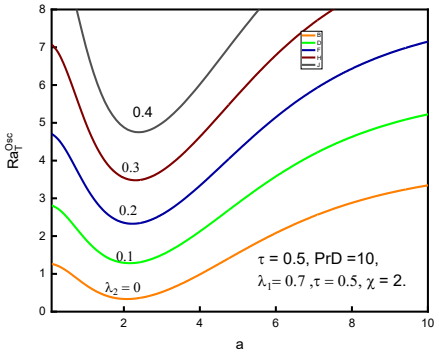


Fig 4. Neutral curves for oscillatory mode for different values of strain retardation time λ_2.

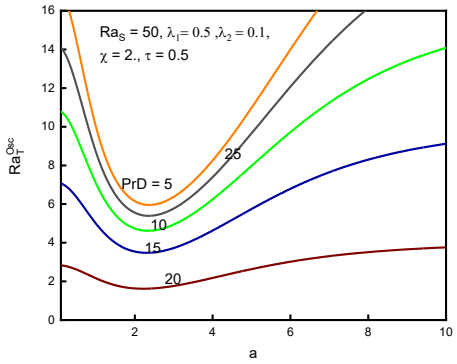


Fig 5. Neutral curves for oscillatory mode for different values of Pr_D.

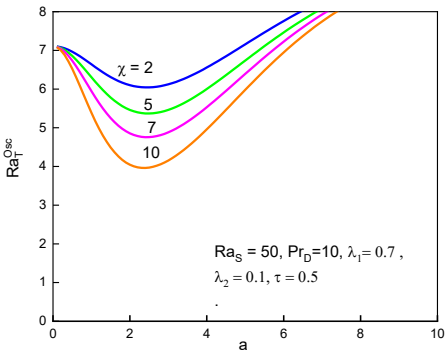


Fig 6. Neutral curves for oscillatory mode for different values of chemical reaction χ.

Hence, larger stress relaxation times delay the onset of oscillatory convection, exhibiting a stabilizing influence. This behavior reflects the increased viscoelastic resistance of the fluid, which suppresses convective motion.

Figure 4 presents the effect of the strain retardation time (λ_2). As $\lambda_2$increases from 0 to 0.4, the neutral curves shift upward and the critical Rayleigh number increases significantly. This indicates that increasing the strain retardation time stabilizes the fluid layer against oscillatory instability by requiring greater thermal driving for convection to occur. Figure 5 illustrates the effect of the Darcy–Prandtl number on the oscillatory instability. It is observed that increasing $Pr_D$from 5 to 20 lowers the neutral stability curves and decreases the corresponding critical Rayleigh number. Therefore, higher values of Pr_Dpromote the onset of convection and have a destabilizing influence. This behaviour indicates that thermal diffusion becomes relatively weaker compared with momentum diffusion, allowing convective motion to develop at lower thermal Rayleigh numbers. The influence of the chemical reaction parameter $\chi$is shown in Figure 6. As χincreases from 2 to 10, the neutral curves shift downward and the critical Rayleigh number decreases considerably. Hence, increasing the chemical reaction parameter advances the onset of oscillatory convection. Physically, stronger chemical reactions modify the concentration field and reduce the stabilizing solutal gradient, thereby destabilizing the fluid layer.

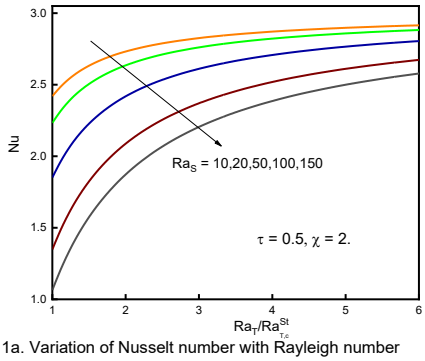


Fig 1a. Variation of Nusselt number with Rayleigh number for different values of Ra_S.

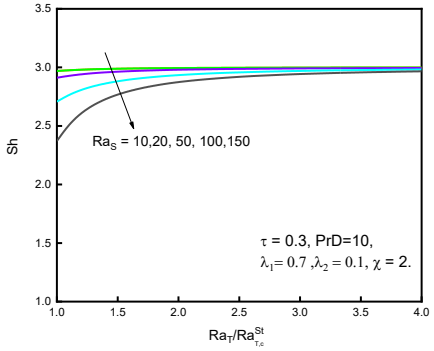
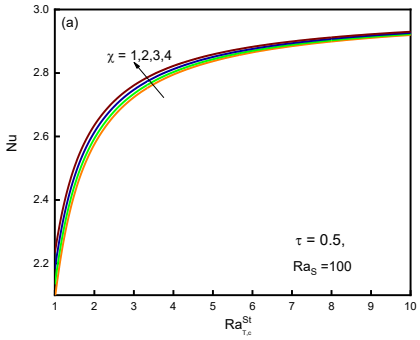


Fig 1b. Variation of Shrwood number with Rayleigh number for different values of Ra_S.

In Fig.1a,b shows the variation of the Nusselt number and Sherwood number for different values of the solute Rayleigh number for the steady finite amplitude motions. we observed that these numbers are independent of viscoelastic parameters and coincide with the Newtonian fluid system.



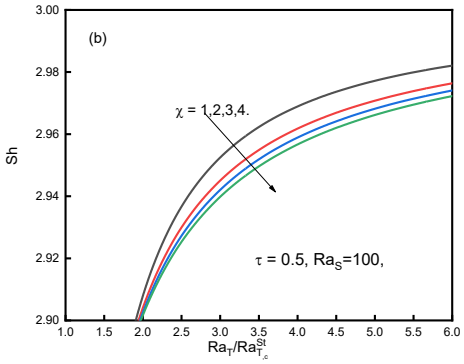


Fig 2a,b. Variation of Nusselt number with Rayleigh number for different values of χ.

The Figs.2a,b shows the variation of the Nusselt number and Sherwood number for different values of the chemical reaction parameter χ for the steady finite amplitude motions. In Fig.2a, we observed the Nusselt number (Nu) increases with the Rayleigh number for all values of chemical reaction parameter χ. Higher values of χ slightly reduce the heat transfer rate, although all curves approach a constant value at larger Rayleigh numbers. Whereas in Fig.2(b) the Sherwood number (Sh) also increases with the Rayleigh number and gradually reaches a steady value. Increasing χslightly decreases the Sherwood number, indicating a small reduction in mass transfer.

VI. CONCLUSIONS

The present investigation demonstrates that the onset of oscillatory double-diffusive convection in a viscoelastic porous medium is strongly influenced by the governing rheological and transport parameters. The strain retardation time (λ_2) and the solute Rayleigh number (Ra_s) exhibit stabilizing effects by increasing the critical thermal Rayleigh number and delaying the onset of convection. Conversely, increasing the Darcy–Prandtl number (Pr_D) and the chemical reaction parameter (χ) lowers the critical Rayleigh number, thereby destabilizing the system. The solute diffusivity ratio (τ) exerts only a comparatively weak influence on the critical conditions. These results demonstrate that viscoelastic properties and chemical reactions play significant roles in controlling the stability characteristics of double-diffusive convection in porous media and provide useful insights for applications involving non-Newtonian fluids, geothermal systems, and chemical transport processes.

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